

NOTA PADAT ADMATH !!

@learalea

Fungsi $x = \text{objek}$
 $f(x) = \text{imej}$

domain = x
 kodomain = $y / f(x)$
 ada tanda \subseteq

ujian garis ① mencancang
 → tentukan sama ada
 suatu graf ialah
 suatu fungsi

Julat = nilai $y / f(x)$ yg
 ada pair dgn x

2 titik = bukan fungsi
 1 titik = fungsi

Fungsi Kuadratik

→ penyempurnaan kuasa 2
 → untuk dapatkan bentuk vertex

syarat
 $a = 1$
 $a = \text{posit.}$

② mengufuk
 → tentukan graf ada
 songsangan
 2 titik \times 1 titik \checkmark

how? $f(x) = ax^2 + bx + c$

Fungsi tak tertakrif
 → penyebut = 0
 $f(x) = \frac{6}{7-7}$

$$= x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$= a(x+p)^2 + q$$

titik maks/min $(-p, q)$

Fungsi nilai mutlak

$$f(x) = |x - 3|$$

graf nilai mutlak
 → ① buat kotak utk
 cari nilai
 ② cari pint -y

punca² persamaan kuadratik
 usually $x = \alpha, x = \beta$

HTP = $\alpha + \beta = -\frac{b}{a}$
 HOP = $\alpha\beta = \frac{c}{a}$

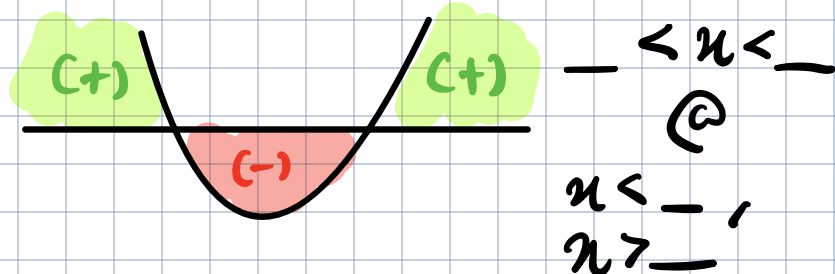
persamaan k.
 $x^2 - (\text{HTP})x + \text{HOP} = 0$

bentuk \vee if $y = mx + c$
 bentuk \wedge if x^2

Penyelesaian ketaksamaan x^2
 kaedah lakaran graf

Fungsi ① biskret integer
 ② selanjur ada c.)

$$fg(x) = gf(x) = x$$



fungsi songsang, left y

Fungsi u^2

pembelajaran

$$\text{U} = b^2 - 4ac > 0$$

punca² nyata

$$\text{U} = b^2 - 4ac = 0$$

1 punca

$$\text{U} = b^2 - 4ac < 0$$

tiada punca nyata

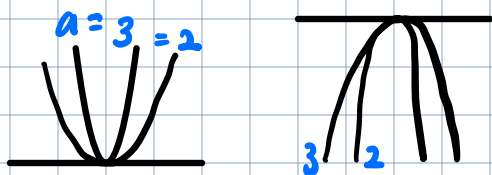
lakaran graf

$$a > 0 \quad \cup$$

$$a < 0 \quad \cap$$

Paksi simetri $u =$ _____

Pembinaan a



@learners

tukar $12.645645645\dots$

$$A = 12.645645645\dots$$

$$A = 12 + N$$

$$N = 0.645645645\dots \quad \text{--- (1)}$$

$$1000N = 645.645645\dots \quad \text{--- (2)}$$

$$\text{(2)} - \text{(1)} \quad 999N = 645$$

$$N = \frac{215}{333}$$

$$A = 12 + \frac{215}{333}$$

$$\therefore A = 12 \frac{215}{333}$$

(3) Log $\log_a N = x, N = a^x$
if $a > 0, a \neq 1$

$$\log_a 1 = 0, \log_a a = 1$$

syarat asas log = nilai positif

$$\log_{10} N = x, \text{anti log } x = N$$

log asas $\log_{10} a = \lg a$

$$\text{if } f(x) = a^x, f^{-1}(x) = \log_a x$$

logaritma asas e aka logaritma jati

$$\ln a = \log_e a$$

$$\log 10 = 1$$

$$\ln e = 1$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$10^{\log x} = x$$

Indeks, surd, Log

(1) Indeks \rightarrow must of the hukum add dkt rumus

(2) Hukum surd = no tk nisbah
no nisbah = perpuluhan
no x nisbah = x ulang

$$2\sqrt{a} \times 4\sqrt{b} = 8\sqrt{ab}$$

caution! $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$
 $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$

surd serupa = $\sqrt{0} \rightarrow$ no sama

Janjang

$$S_n = \frac{n}{2} (a + l) \text{ sebutan terakhir}$$

$$T_n = S_n - S_{n-1}$$

$$S_\infty = \frac{a}{1-r}$$

hayil tambah sebutan ke 5 hingga sebutan 10

$$= S_{10} - S_4$$

J. Aritmetik = +/-

sebutan

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

(hasil tambah)

Geometri = ÷ / x

$$T_n = ar^{n-1} \text{ bilq}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad r < 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad r > 1$$

no perpuluhan berulang

$$0.345345345 \dots$$

$$= 0 + 0.345 + 0.000345$$

$$a = 0.345 \quad r = 0.001$$

$$S_\infty = \frac{0.345}{1-0.001}$$

$$= \frac{115}{333}$$

Hukum Linear

garis lurus penyuaian terbaik
→ mesti balance!

Cth: $y = ab^x$ tukar bukan linear → linear
 $\log_{10} y = \log_{10} a + x \log_{10} b$
y C X m

@

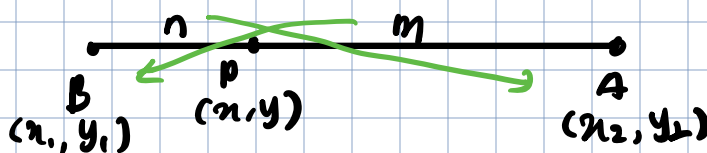
$$y = ax^3 + bx^2$$

$$\frac{y}{x^2} = ax + b$$

$$\frac{y}{x^2} = mx + c$$

pastikan dim
 $y = mx + c$

Geometri Koordinat



$$P = \left(\frac{nx_2 + mx_1}{n+m}, \frac{ny_2 + my_1}{n+m} \right)$$

$$\text{Jarak} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{titik tengah} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

// Selari $m_1 = m_2$
 ⊥ serenjang @ 90° $m_1 m_2 = -1$

luas poligon Δ @ \square

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2} |(\Sigma \text{pink}) - (\Sigma \text{green})|$$

Syarat!
mesti susun titik yg bersebelahan susunan penting!

Persamaan Locus

- ① titik P bergerak dgn keadaan jarak dan titik A adalah 4

$$\sqrt{(x-x_A)^2 + (y-y_A)^2} = 4$$

↳ untuk bulatan

- ② titik S bergerak dgn jarak dan titik A adalah 2 kali ganda dari titik L

$$SA = 2SL$$

$$\sqrt{(x-x_A)^2 + (y-y_A)^2}$$

$$= 2\sqrt{(x-x_L)^2 + (y-y_L)^2}$$

↳ kinda like pembahagi

vektor

tatanda: \vec{AB} , \vec{BA} , \vec{a} in bold

if $\vec{AB} = \vec{a}$, $\vec{BA} = -\vec{a}$
↳ arah PENTING!

vektor = magnitud o
sifar = arah x tahu

selari, $\vec{AB} = k\vec{CD}$

vektor tak selari dan bukan sifar

$$h\vec{a} = k\vec{b}; h = k = 0$$

vektor unit

$$\hat{r} = \frac{x\vec{i} + y\vec{j}}{|\vec{r}|}$$

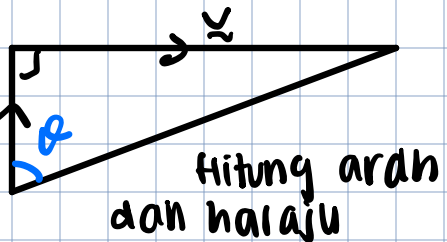
magnitud

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

cth soalan

$$u = 5 \text{ kmj}^{-1}$$

$$\vec{v} = 10 \text{ kmj}^{-1}$$

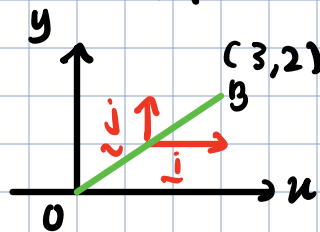


$$\begin{aligned} \text{halaju} &= |\underline{u} + \underline{v}| \\ &= \sqrt{5^2 + 10^2} \\ &= 11.18 \text{ kmj}^{-1} \end{aligned}$$

$$\tan \theta = \frac{10}{5} \quad \theta = 63.43^\circ$$

∴ bergerak pd bearing 63.43° dengan halaju 11.18 kmj^{-1}

Vektor dalam Satah Cartes



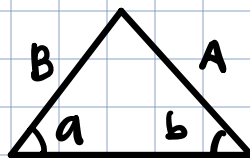
boleh tulis:

$$\textcircled{1} x\vec{i} + y\vec{j} @ \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{OB} = 3\vec{i} + 2\vec{j}$$

"vektor unit" = magnitud = 1

Penyelesaian Δ



petua sinus

$$\frac{\sin a}{A} = \frac{\sin b}{B}$$

kes berambiguiti → bina Δ baru

Petua Kosinus



$$A^2 = B^2 + C^2 - 2BC \cos a$$

luas Δ $\textcircled{1} \frac{1}{2} abs \sin c$ sudut kandung!

Rumus Heron

$$\textcircled{1} s = \frac{a + b + c}{2}$$

$$\textcircled{2} \sqrt{s(s-a)(s-b)(s-c)}$$

Number Indeks

$$I = \frac{Q_1}{Q_0} \times 100$$

Q_0 ← tahun asas

Indeks gubahan, $\bar{I} = \frac{\sum IW}{\sum W}$ ← pemberat

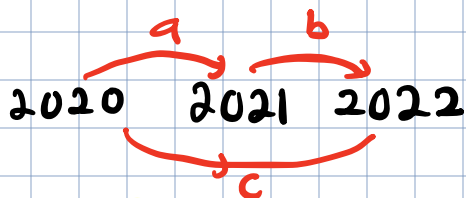
$I = 100$ kekal
meningkat 20 %

$$I = 120$$

menurun 20 %

$$I = 80$$

if ada 3 tahun



$$c = \frac{ab}{100}$$

Sukatan membulat

$$180^\circ = \pi$$

$$360^\circ = 2\pi$$

luas sektor = $\frac{1}{2} r^2 \theta$ ^{in rad}

panjang lengkok = $r\theta$

luas tembereng

$$= \frac{1}{2} r^2 (\theta - \sin \theta)$$

rad & degree

* akan guna teorem
juga utk bab ni

Pembezaan

$\frac{dy}{dx}$ guna Prinsip pertama

$$\text{cth: } y = 5x - 4 \quad \text{--- (1)}$$

$$y + \delta y = 5(x + \delta x) - 4$$

$$y + \delta y = 5x + 5\delta x - 4 \quad \text{--- (2)}$$

$$\text{(2) - (1)} \quad \frac{\delta y}{\delta x} = \frac{5\delta x}{\delta x}$$

$$\frac{dy}{dx} = \frac{\delta y}{\delta x} \lim_{\delta x \rightarrow 0}$$

$$\frac{dy}{dx} = 5$$

Pembezaan peringkat pertama $\frac{dy}{dx}$

→ nak cari m

$$y = 5x^2 + 10x \quad \frac{dy}{dx} = 10x + 10$$

$$y = a(bx + c)^n$$

$$\frac{dy}{dx} = na(bx + c)^{n-1} (b)$$

Peringkat kedua $\frac{d^2y}{dx^2}$

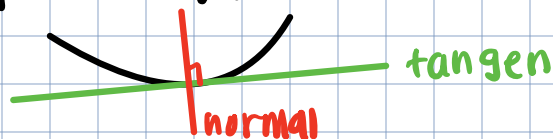
→ nak tentukan

jenis titik maks / min

$$\frac{dy}{dx} = 10x + 10$$

$$\frac{d^2y}{dx^2} = 10$$

$$m_{\text{normal}} (m_{\text{tangen}}) = -1$$



titik pusingan $\frac{dy}{dx} = 0$

if $\frac{d^2y}{dx^2} > 0$ titik min, $\frac{d^2y}{dx^2} < 0$ titik maks

Kadar perubahan

$$y = \frac{dy}{dt}$$

cairi kadar perubahan

$$\frac{dn}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

Perubahan kecil

$$\Delta y \approx \frac{dy}{dx} \times \Delta x$$

penghampiran @
nilai = $y_{ori} + \Delta y$
hampir

Peratus perubahan

① dalam $x = \frac{\Delta x}{x} \times 100\%$

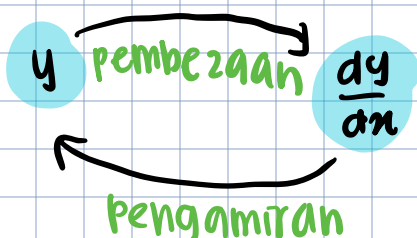
② dalam $y = \frac{\Delta y}{y} \times 100\%$

Pengamiran

① tak tentu $\int a x^n dx = \frac{a x^{n+1}}{n+1} + c$ wajib ada!

- $\int a(bx+c)^n dx = \frac{a(bx+c)^{n+1}}{(n+1)b}$

② tentu \int_a^b ada nilai



luas bawah lengkung @ garis

$$\int_a^b y dx \quad \begin{array}{|c|} \hline (+) \\ \hline (-) \\ \hline \end{array}$$

$$\int_a^b x dy \quad \begin{array}{|c|} \hline (-) \\ \hline (+) \\ \hline \end{array}$$

so if dkt (-) kena letak
 $|y|$ utk positifkan dkt pengiraan

isipadu janaan

$$\pi \int_a^b y^2 dx \quad / \quad \pi \int_a^b x^2 dy$$

lam²:

$$\int_a^b k f(x) dx = k \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

Pilih atur dan gabungan

Penting susunan
"bil cara"

① PA $n! = {}^n P_n = n(n-1)(n-2) \dots$

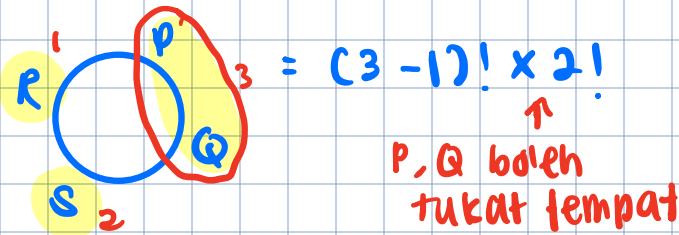
secaman = $\frac{n!}{a!b!}$

di susun bulat (meja) = $\frac{(n-1)!}{2}$
mahik = $\frac{(n-1)!}{2}$ ← syarat n kena gung semua

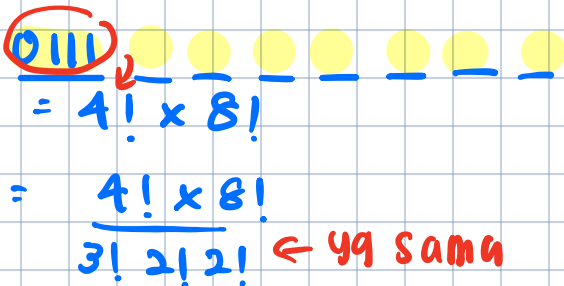
Pilih statik = $\frac{nPr}{r}$

rantai / manik = $\frac{nPr}{2r}$

cth P, Q, R, S duduk semeja tapi P, Q mesti bersebelahan



② Huruf OPTIMISTICS if vokal duduk sekali



② Gabungan

${}^n C_r = \frac{n!}{(n-r)! r!}$

taburan kebarangkalian

① binomial $X \sim B(n, p)$

p = berjaya p + q = 1

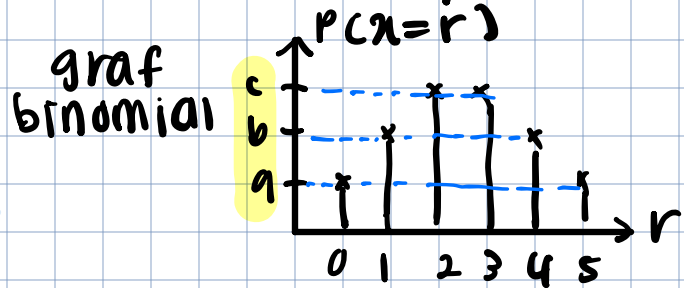
Per cubaan Bernoulli

→ hanya 2 kejudahan dalam diskret cini
 $x = \{0, 1\}$

${}^n C_n = 1$ ${}^n C_0 = 1$

① taburan binomial

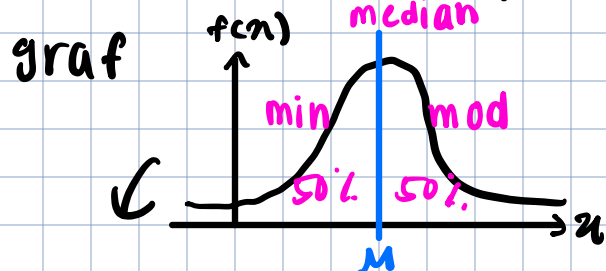
data { min = np, mod, median, $\sigma = \sqrt{npq}$, $\sigma^2 = npq$ }



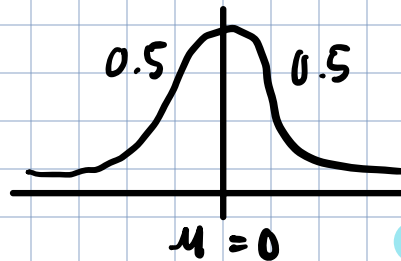
$a + b + c = 1$

$x = \{0, 1, 2, 3, 4, 5\}$

② Normal $X \sim N(\mu, \sigma^2)$
 → data selanjur

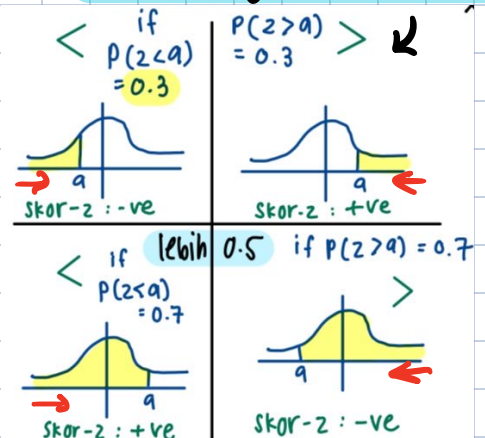


tukar to skor-z



$z = \frac{x - \mu}{\sigma}$

kebarangkalian < 0.5



→ "mula di mana!"

Fungsi Trigonometri

SPM NOTES - faarah
@lycaniv

Sudut tetap yg beza:

[nilai (+) / (-)] II

$$90^\circ < \theta \leq 180^\circ$$

sin (+)

tan (+)

III

$$180^\circ < \theta \leq 270^\circ$$

all (+)

Sudut I

$$0 < \theta \leq 90^\circ$$

kos

Sudut IV

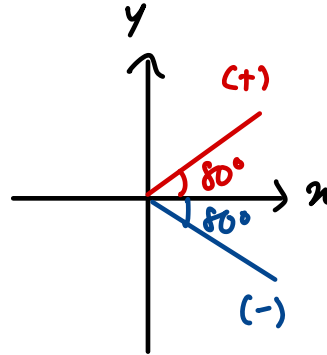
$$270^\circ < \theta \leq 360^\circ$$

All saya tak kesah!

Selesaikan
Lukiskan
Buktikan

Sudut (+) → lawan arah jam

Sudut (-) → ikut arah jam



Nisbah Trigonometri

SOH CAH TOA → asal

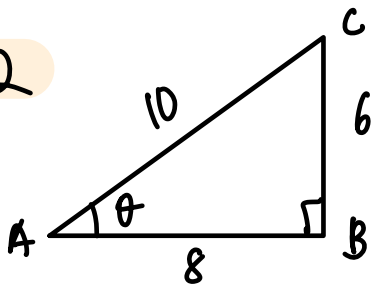
$$\text{kosek } \theta \rightarrow \frac{1}{\sin \theta} / \frac{H}{O}$$

$$\text{sek } \theta \rightarrow \frac{1}{\cos \theta} / \frac{H}{A}$$

$$\text{kot } \theta \rightarrow \frac{1}{\tan \theta} / \frac{A}{O}$$

Contoh 2

Cari nilai



$$a) \text{ kosek } \theta = \frac{10}{6}$$

$$b) \text{ kot } \theta = \frac{8}{6}$$

$$b) \text{ sek } \theta = \frac{10}{8}$$

Contoh 3

$$\alpha = 56^\circ$$

$$a) \text{ kosek } 56^\circ = \frac{1}{\sin 56^\circ} = 1.206$$

$$b) \text{ sek } 56^\circ = \frac{1}{\cos 56^\circ} = 1.788$$

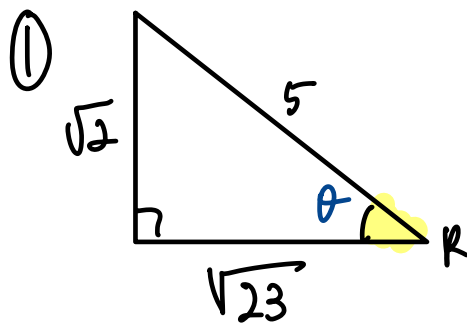
Sudut pelengkap $A + B = 90^\circ$

penggenap $A + B = 180^\circ$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Latihan sendiri 6.2



a) $\cot R = \frac{\sqrt{23}}{\sqrt{2}}$

b) $\sin^2 R = \left(\frac{\sqrt{2}}{5}\right)^2 = \frac{2}{25}$

c) $\frac{\cos R - \sin R}{\operatorname{cosec} R}$

$$\frac{\frac{\sqrt{23}}{5} - \frac{\sqrt{2}}{5}}{\frac{5}{\sqrt{2}}} = \frac{\frac{\sqrt{23} - \sqrt{2}}{5}}{\frac{5}{\sqrt{2}}}$$

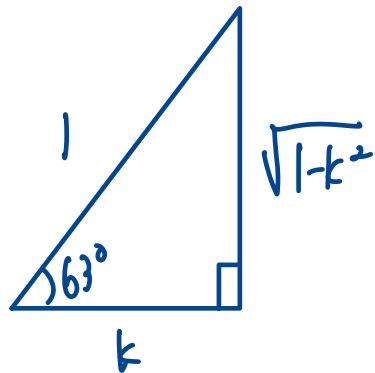
$$= \frac{\sqrt{23} - \sqrt{2}}{5} \times \frac{\sqrt{2}}{5}$$

$$= \frac{\sqrt{46} - 2}{25}$$

Contoh 5

$\cos 63^\circ = k, k > 0$

$\cos 63^\circ = \frac{k}{1}$



a) $\sin 63^\circ = \sqrt{1-k^2}$

b) $\sin 27^\circ = \cos(90-27^\circ) = \cos 63^\circ = k$

c) $\operatorname{cosec} 27 = \sec(90-27^\circ) = \sec 63^\circ = \frac{1}{\cos 63^\circ} = \frac{1}{k}$

↖ cosec = sek

Sudut pelengkap

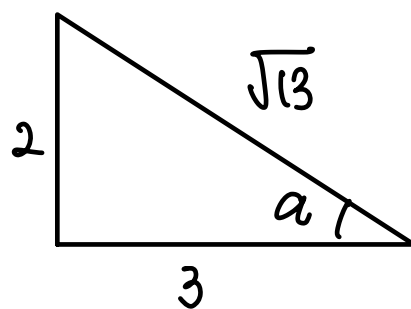
$\cos \theta = \sin(90-\theta)$

$\sin \theta = \cos(90-\theta)$

$\tan \theta = \cot(90-\theta)$

$\cot \theta = \tan(90-\theta)$

②



a) $\sin a = \frac{2}{\sqrt{13}}$

e) $\frac{4 - \sec^2 a}{2 - \sec^2 a} = \frac{4 - \left(\frac{\sqrt{13}}{3}\right)^2}{2 - \left(\frac{\sqrt{13}}{3}\right)^2}$

$$= \frac{36-13}{9} \times \frac{3}{6-\sqrt{13}}$$

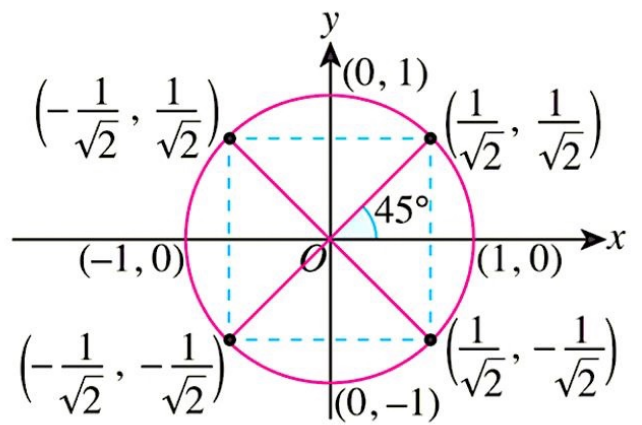
$$= \frac{69}{9(6-\sqrt{13})} = \frac{23}{3(6-\sqrt{13})}$$

tentukan nilai ① guna calc

② gunakan bulatan unit

$$\sin \theta = y \quad \cos \theta = x$$

$$\tan = \frac{y}{x}$$



③ guna nilai nisbah trigo sudut rujukan yg sepadan

$\sin (+)$ II → kosek	$\text{All } (+)$ I
$\tan (+)$ III → kot	$\cos (+)$ IV → sek

- Sukuan
- I = θ
 - II = $180^\circ - \theta$
 - III = $\theta - 180^\circ$
 - IV = $180^\circ + \theta$
 - IV = $360^\circ - \theta$

Contoh 8

$$\sin 30^\circ = 0.5$$

$$\cos 30^\circ = 0.866$$

$$\text{sukuan II} = -\frac{1}{0.866}$$

$$\cos (-) = -1.155$$

a) sek 150° rujukan = $-\text{sek } 30^\circ = -\frac{1}{\cos 30^\circ}$

④ sudut² khas
 guna segi tiga bersudut tegak
 $30^\circ, 45^\circ, 60^\circ$

Contoh 9

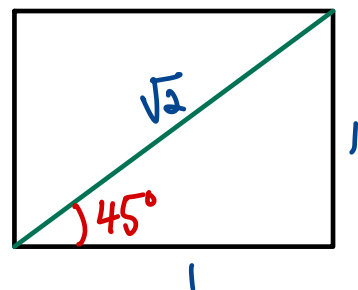
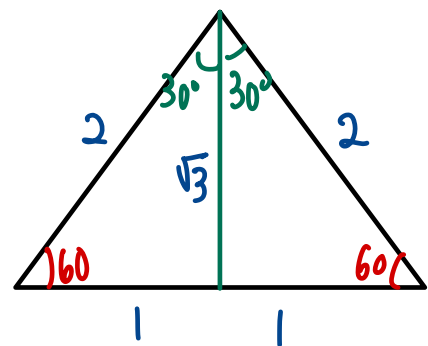
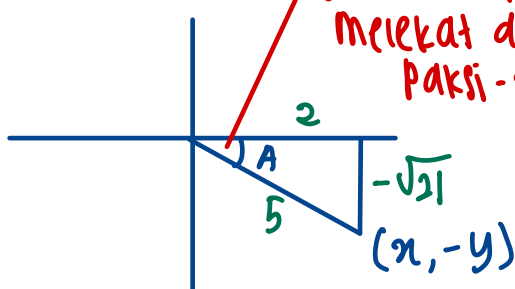
$$\cos A = \frac{2}{5}, \quad 270^\circ \leq A \leq 360^\circ$$

Sukuan I & IV

Sukuan IV

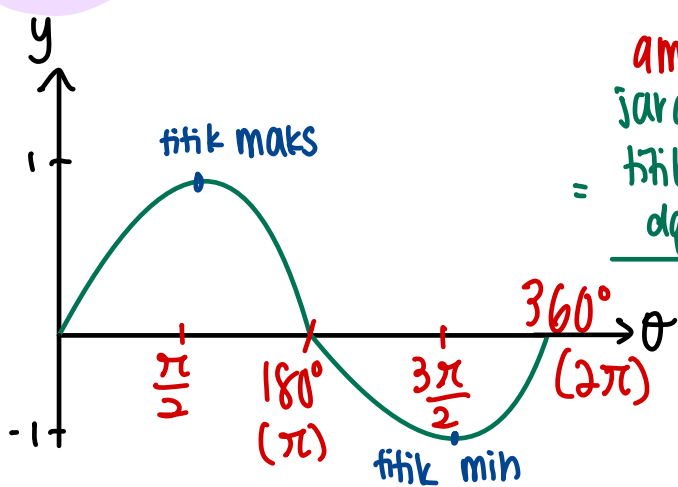
sudut akan melekat dgn paksi-x

graf



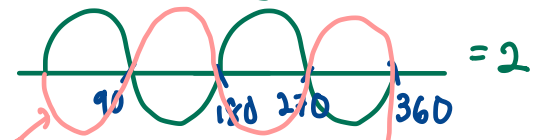
graf fungsi

① sinus, $y = \sin \theta \rightarrow y = a \sin b\theta + c$ — permulaan dkt paksi-y



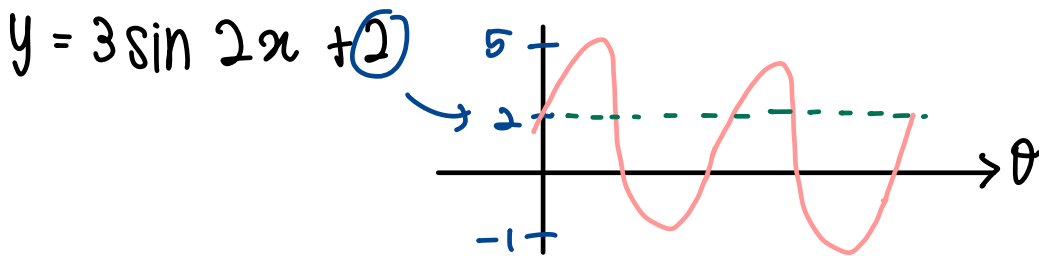
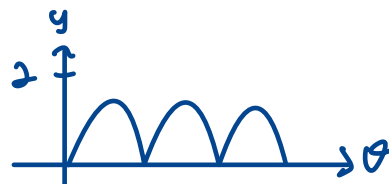
amplitud
jarak antara
titik maks
dgn min
= $\frac{2}{2}$

nilai kitaran
dalam 360°



kalo (-) dpt persamaan
cth: $-a \sin 2x$

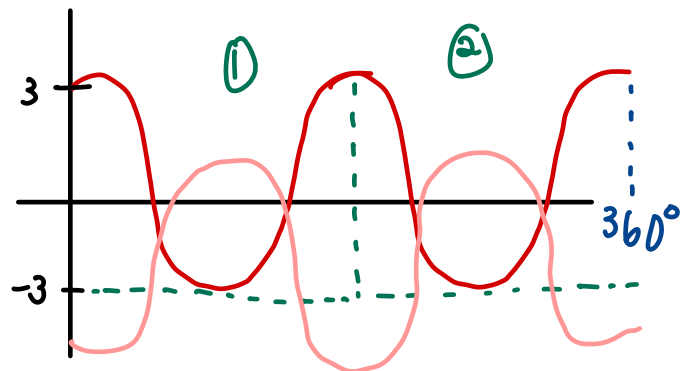
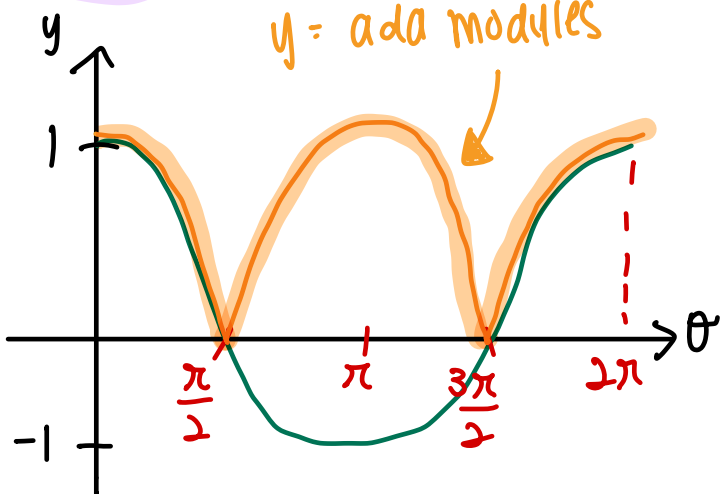
kalo ada modulus, $y = |-2 \sin \frac{3}{2} x|$
akan jadi positif
yg ddn (+) tk payah ubah



② kosinus, $y = a \cos b\theta + c$

ada when nilai paksi-y tak
balance y: kalo (-)

cth: $y = 3 \cos 2x$



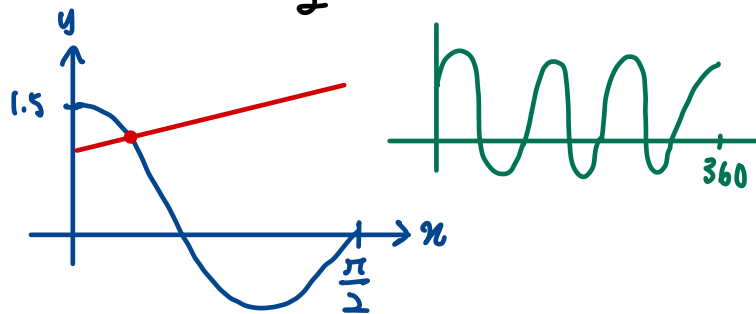
Pengelasan masalah

Kendiri 6.5

③ carilah graf $y = \frac{3}{2} \cos 3x$, $y = \frac{x}{\pi} + 1$

julat $0 \leq x \leq \frac{\pi}{2}$

asal:



$$y = \frac{x}{\pi} + 1$$

x	0	$\frac{\pi}{2}$
y	1	$\frac{3}{2}$

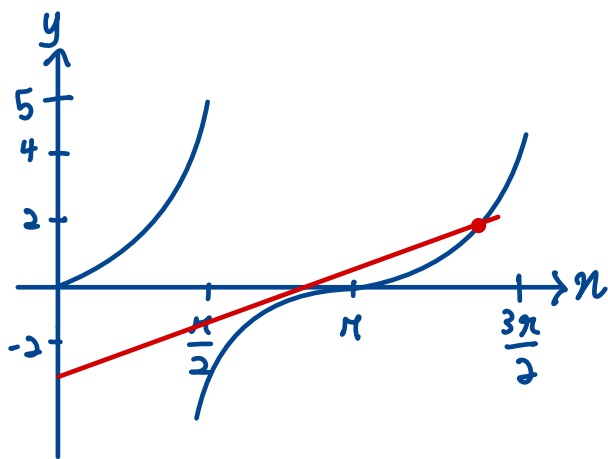
Bil penyelesaian = 1

Formatif 6.3

② lukis graf $y = 5 \tan x$ $0 \leq x \leq \frac{3\pi}{2}$

lukis garis lurus bersesuaian

$$30 \tan x - 6x + 5x = 0$$



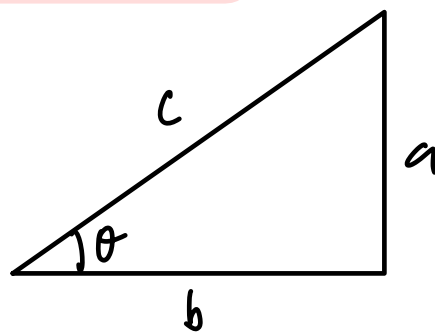
$$6(5 \tan x - x + \frac{5}{6}x) = 0$$

$$y - x + \frac{5}{6}x = 0, y = x - \frac{5x}{6}$$

x	0	$\frac{3\pi}{2}$
y	$-\frac{5}{6}x$	$\frac{2}{3}x$

bilangan penyelesaian = 1

Identiti ASAS



$$a^2 + b^2 = c^2$$

$$\begin{aligned} \sin^2 45 &= \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

b)

$$\begin{aligned} &\frac{1 - \sin^4 A}{\cos^4 A} \\ &= \frac{(1 + \sin^2 A)(1 - \sin^2 A)}{(\cos^2 A)^2} \\ &= \frac{1 + \sin^2 A (\cancel{\cos^2 A})}{(\cos^2 A)^2} \\ &= \frac{1 + \sin^2 A}{\cos^2 A} \\ &= \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} \\ &= \sec^2 A + \tan^2 A \\ &= (1 + \tan^2 A) + \tan^2 A \\ &= 1 + 2 \tan^2 A \end{aligned}$$